Module 2- GEARS

Lecture – 11 HELICAL GEARS

Contents

11.1 HELICAL GEARS – an introduction

In spur gears Fig.11.1 dealt earlier, the teeth are parallel to the axis whereas in helical gears Fig.11.2 the teeth are inclined to the axis. Both the gears are transmitting power between two parallel shafts.

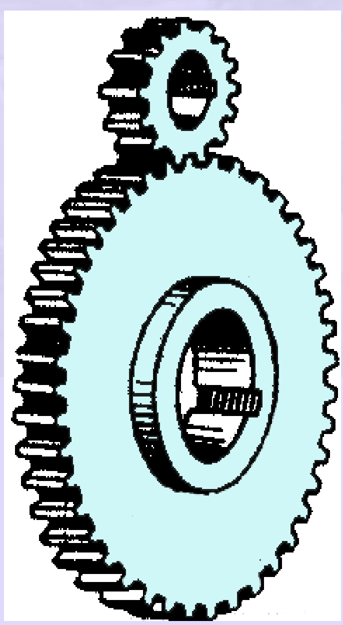

Fig.11.1 Spur gear

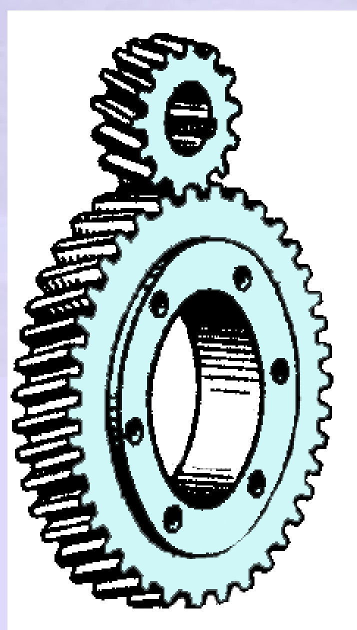

Fig.11.2 Helical gear

Helical gear can be thought of as an ordinary spur gear machined from a stack of thin shim stock, each limitation of which is rotated slightly with respect to its neighbours as in Fig.11.3. When power is transmitted both shafts are subjected to thrust load on the shaft.

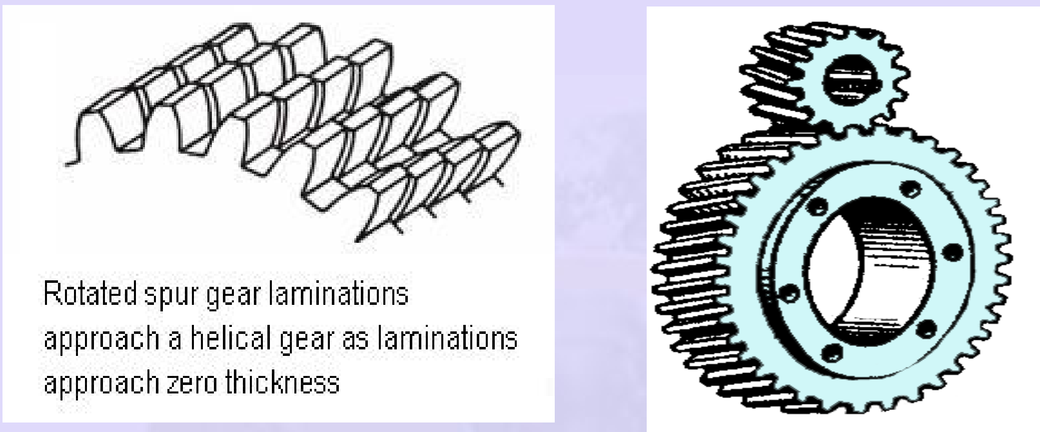

Fig.11.3 Illustration of concept of helical gear

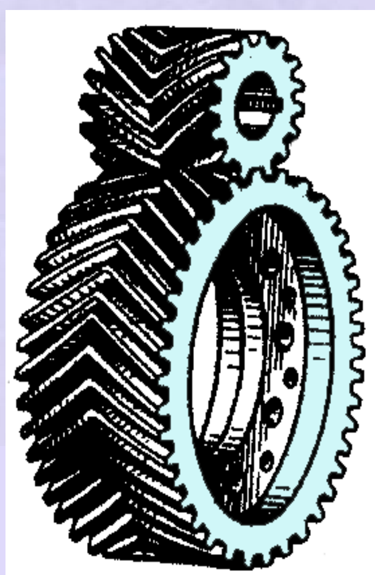

Fig.11.4 Double helical gear or herringbone gear

Herringbone or double helical gear shown in Fig. 11.4 can be two helical gears with opposing helix angle stacked together. As a result, two opposing thrust loads cancel and the shafts are not acted upon by any thrust load.

The advantages of elimination of thrust load in Herringbone gears, is obliterated by considerably higher machining and mounting costs. This limits their applications to very heavy power transmission.

(c) *Double helical* or *herringbone* gears may or may not have a center space, depending on manufacturing method.

Fig.11.5 Double helical gear of a cement mill rotary gear drive

Fig. 11.6 Crossed helical gears.

Crossed helical gears As in Fig. 11.6 are used for transmitting power between two non-parallel, non-intersecting shafts. Common application is distributor and pump drive from cam shafts in automotive engines.

11.2 HELICAL GEARS- KINEMATICS

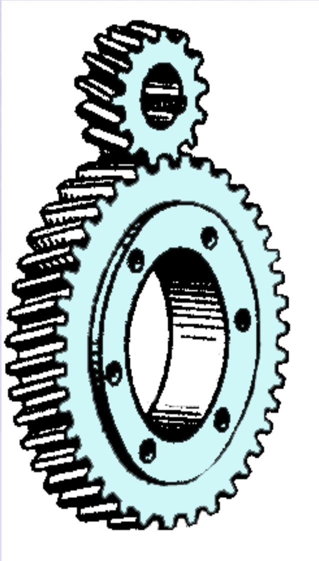

Fig.11.7 Helical gear

When two helical gears are engaged as in the Fig. 11.7, the helix angle has to be the same on each gear, but one gear must have a right-hand helix and the other a left-hand helix.

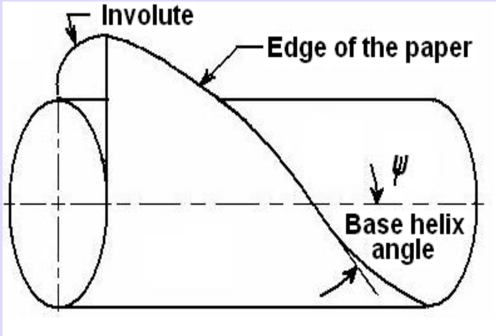

Fig.11.8 Illustration of helical gear tooth formation

The shape of the tooth is an involute helicoid as illustrated in the Fig. 11.8. If a paper piece of the shape of a parallelogram is wrapped around a cylinder, the angular edge of the paper becomes the helix. If the paper is unwound, each point on the angular edge generates an involute curve. The surface got when every point on the edge generates an involute is called involute helicoid. In spur gear, the initial contact line extends all the way across the tooth face. The initial contact of helical gear teeth is point which changes into a line as the teeth come into more engagement.

In spur gears the line contact is parallel to the axis of rotation; in helical gear the line is diagonal across the face of the tooth. Hence gradual engagement of the teeth and the smooth transfer of load from one tooth to another occur.

This gradual engagement makes the gear operation smoother and quieter than with spur gears and results in a lower dynamic factor, K_v. Thus, it can transmit heavy loads at high speeds. Typical usage is automotive transmission for compact and quiet drive.

11.3 HELICAL GEARS – GEOMETRY AND NOMENCLATURE

The helix angle ψ, is always measured on the cylindrical pitch surface Fig. 11.8. ψ value is not standardized. It ranges between 15^o and 45^o. Commonly used values are 15, 23, 30 or 45^o. Lower values give less end thrust. Higher values result in smoother operation and more end thrust. Above 45^o is not recommended.

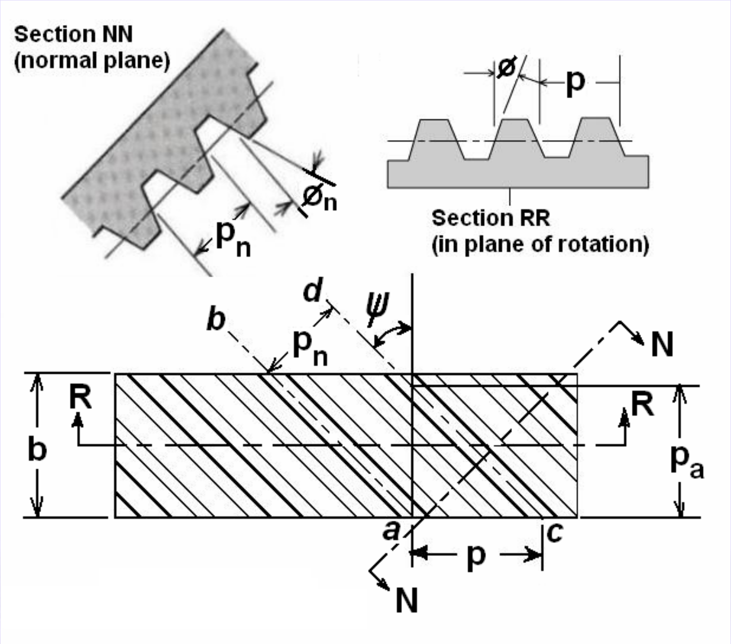

Fig.11.9 Portion of helical rack

The circular pitch (p) and pressure angle (Ø) are measured in the plane of rotation, as in spur gears. These quantities in normal plane are denoted by suffix n (p_n, $Ø_n$) as shown in Fig. 11.9.

From geometry we have normal pitch as

$$p_n = p \cos \psi \qquad (11.1)$$

Normal module m_n is

$$m_n = m \cos \psi \qquad (11.2)$$

m_n is used for hob selection.

The pitch diameter (d) of the helical gear is:

$$d = Z\,m = Z\,m_n / \cos \psi \qquad (11.3)$$

The axial pitch (p_a) is:

$$p_a = p / \tan \psi \qquad (11.4)$$

For axial overlap of adjacent teeth, $b \geq p_a$ **(11.5)**

In practice $b = (1.15 \sim 2)\, p_a$ is used.

The relation between normal and transverse pressure angles is

$$\tan \varnothing_n = \tan \varnothing \,.\cos \psi \qquad (11.6)$$

In the case of helical gear, the resultant load between mating teeth is always perpendicular to the tooth surface. Hence bending stresses are computed in the normal plane, and the strength of the tooth as a cantilever beam depends on its profile in the normal plane. Fig. 11.10 shows the view of helical gear in normal and transverse plane.

The following figure shows the pitch cylinder and one tooth of a helical gear. The normal plane intersects the pitch cylinder in an ellipse.

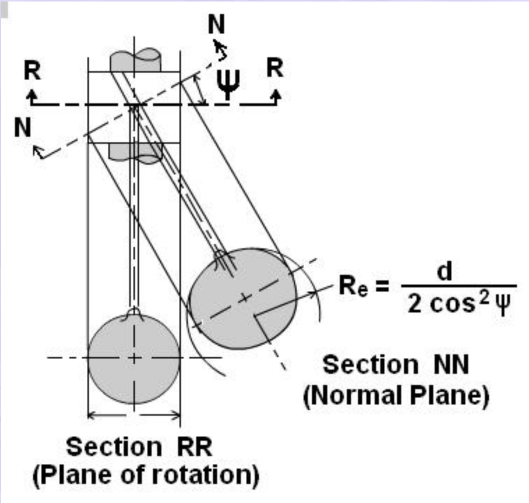

Fig.11.10 View of helical gear in normal and transverse sections

The shape of the tooth in the normal plane is nearly the same as the shape of a spur gear tooth having a pitch radius equal to radius R_e of the ellipse.

$$R_e = d/\,(2\cos^2 \psi) \qquad (11.7)$$

The equivalent number of teeth (also called virtual number of teeth), Z_v, is defined as the number of teeth in a gear of radius R_e:

$$Z_v = \frac{2R_e}{m_n} = \frac{d}{m_n \cos^2 \psi} \qquad (11.8)$$

Substituting $m_n = m \cos\psi$, and $d = Z m$

$$Z_v = \frac{Z}{\cos^3 \psi} \qquad (11.9)$$

When we compute the bending strength of helical teeth, values of the Lewis form factor Y are the same as for spur gears having the same number of teeth as the virtual number of teeth (Z_v) in the Helical gear and a pressure angle equal to $Ø_n$.

Determination of geometry factor J is also based on the virtual number of teeth. These values are plotted in Fig.11.11 and 11.12.

HELICAL GEARS – GEOMETRY FACTOR

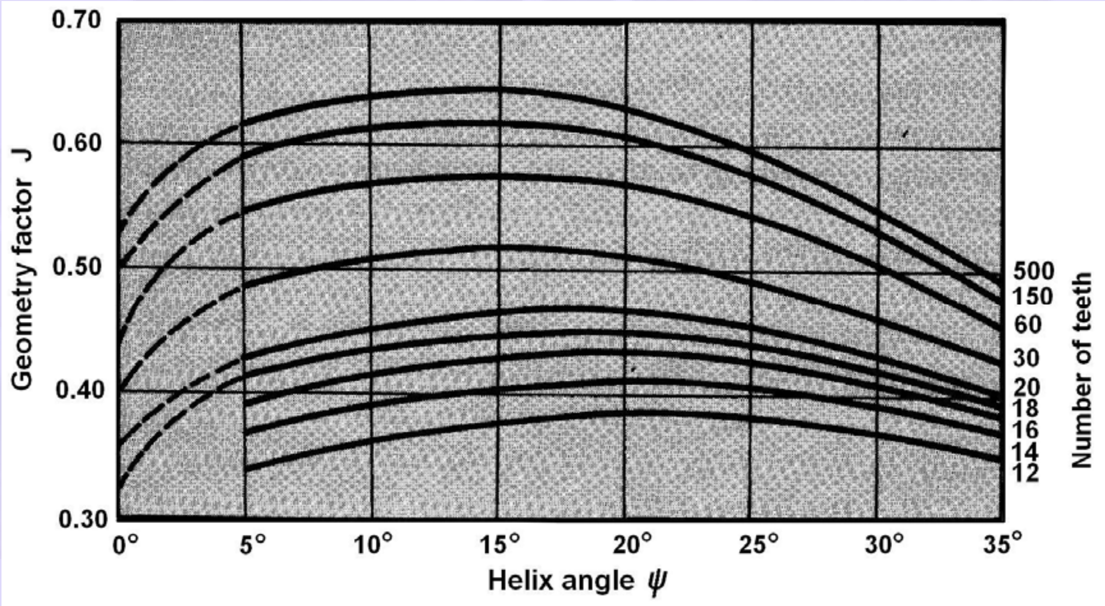

Fig 11.11 Geometry factor for use with a 75-tooth mating gear, pressure angle (Φ_n) 20^o, std. addendum of 1m and shaved teeth

HELICAL GEARS – J FACTOR MULTIPLIERS

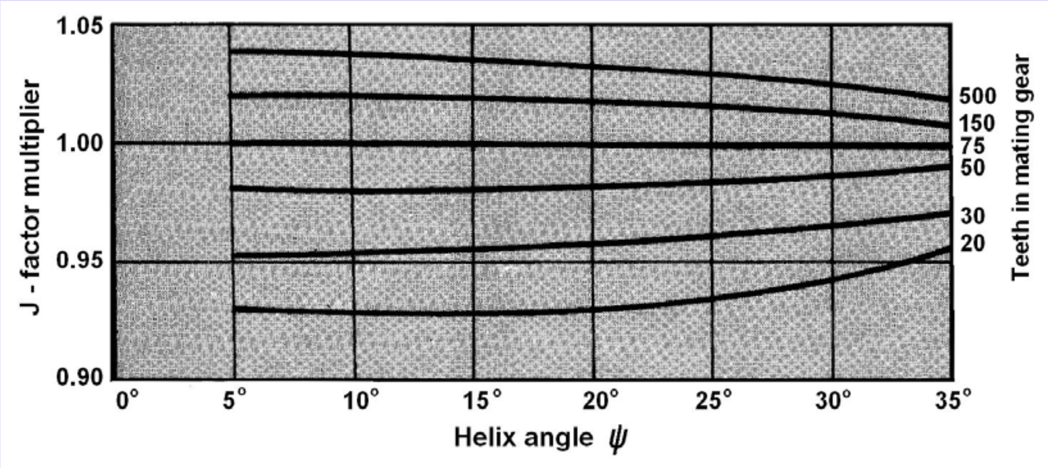

Fig.11.12 J factor multipliers to be used with mating gears other than 75 teeth

11.4 HELICAL GEARS - FORCE ANALYSIS

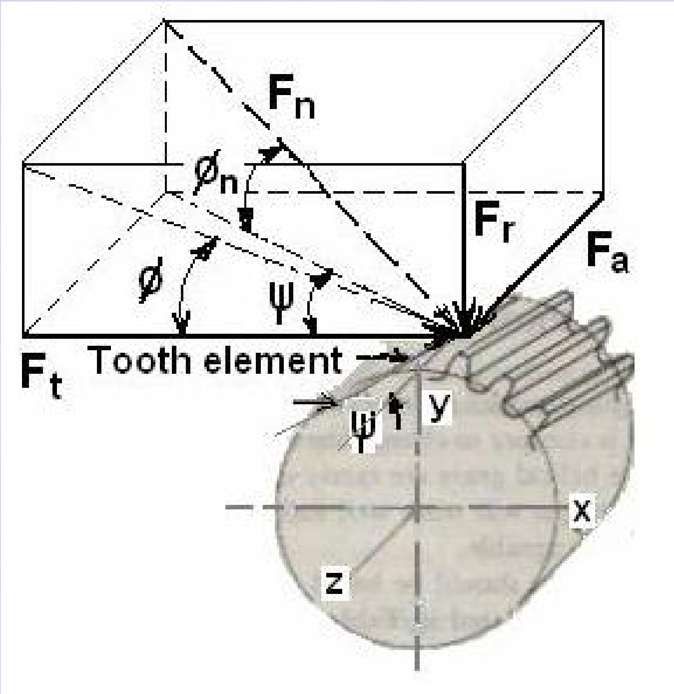

Fig.11.13 Tooth force acting on a right hand helical gear

3-dimensional view of the forces acting on a helical gear tooth is shown in the Fig.11.13. Resolving F_n

$F_r = F_n \sin Ø_n$ (11.10)

$F_t = F_n \cos Ø_n \cos \psi$ (11.11)

$F_a = F_n \cos Ø_n \sin \psi$ (11.12)

$F_r = F_t \tan Ø$ (11.13)

$F_a = F_t \tan \psi$ (11.14)

$$F_n = \frac{F_t}{\cos\varphi_n \cos\psi} \quad (11.15)$$

Fig.11.14 illustrates the tooth forces acting on spur and helical gears. For spur gears, the total tooth force consists of components tangential F_t and radial F_r forces. For helical gears, component F_a is added and normal section NN is needed to show a true view of total tooth force F_n.

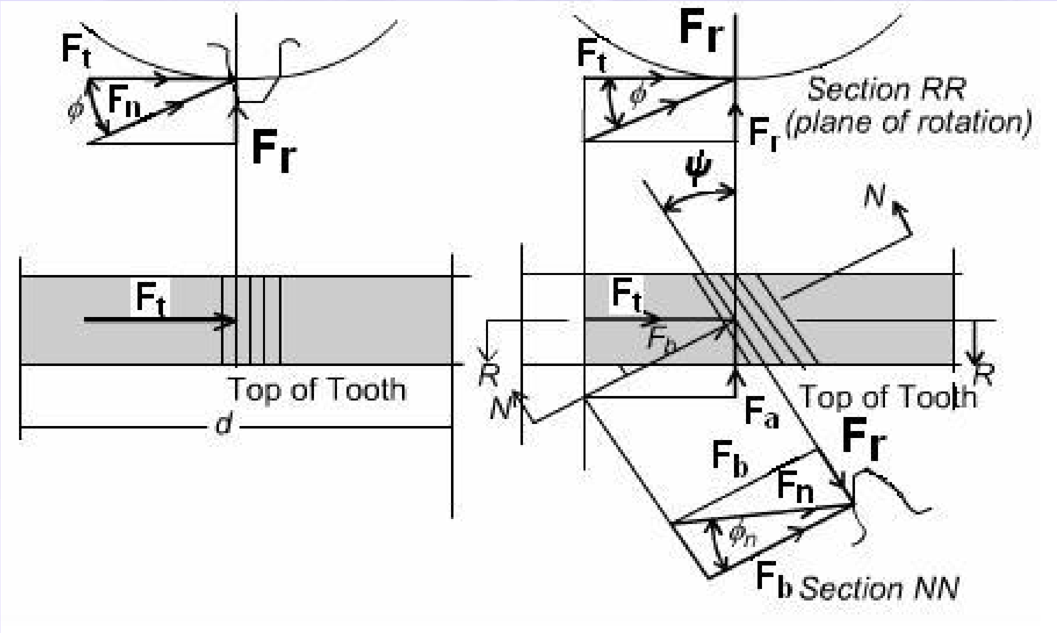

Fig. 11.14 The comparison of force components on spur and helical gears

The vector sum F_t and F_a is labeled F_b; the subscript b being chosen because F_b is the bending force on the helical tooth (just as F_t is bending force on the spur tooth).

The force component associated with power transmission is only F_t

$$F_t = \frac{1000W}{V} \qquad (11.16)$$

Where F_t is in (N), W is in kW, and V is the pitch line velocity in (m/s).

$$F_b = F_t / \cos\psi \qquad (11.17)$$

$$F_r = F_b \tan \phi_n \qquad (11.18)$$

$$F_r = F_t \tan \phi \qquad (11.19)$$

Combining equation 11.12, 11.17 and 11.18

$$\tan \phi_n = \tan \phi \cos \psi \qquad (11.20)$$

11.5 HELICAL GEAR - TOOTH BENDING STRESS

The bending stress equation for helical gear teeth is given as

$$\sigma = \frac{F_t}{b\,m\,J} K_v K_o (0.93 K_m) \qquad (11.21)$$

Introduction of constant 0.93 with the mounting factor reflects slightly lower sensitivity of helical gears to mounting conditions. The J factor can be determined from Figs.11.15 and 1.16.

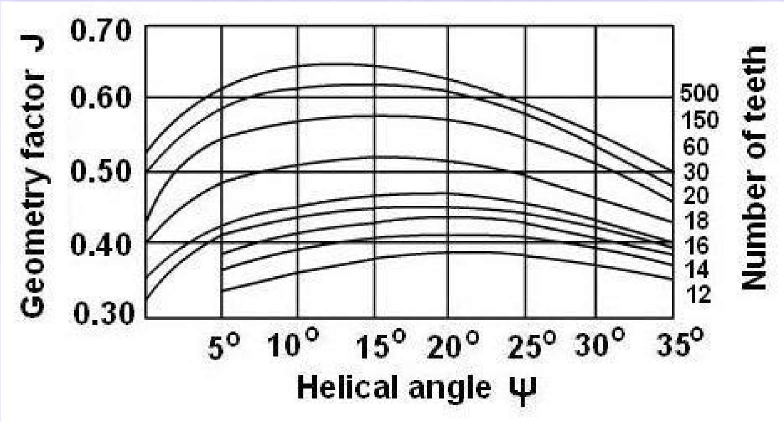

Fig.11.15 Geometry factor J for helical gear with $\varphi_n = 20^o$ and mating with 75 tooth gear

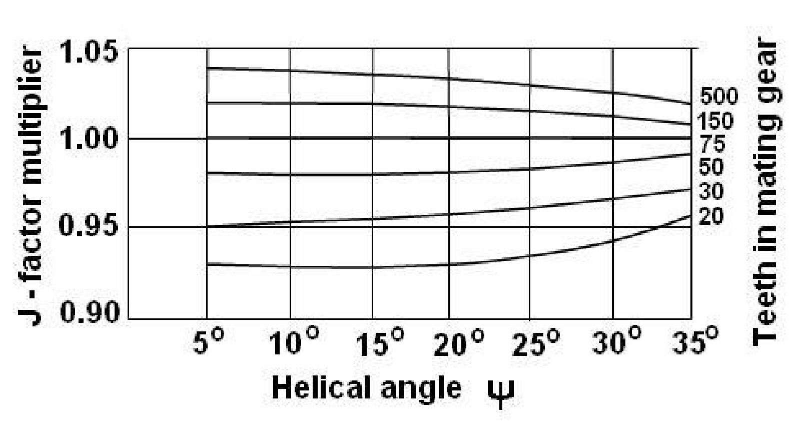

Fig.11.16 J-factor multiplier when the mating gear has tooth other than 75

Velocity factor K_v is calculated from the equation 11.22 or from Fig.11.17

$$K_v = \left[\frac{78 + (200V)^{0.5}}{78}\right]^{0.5} \qquad (11.22)$$

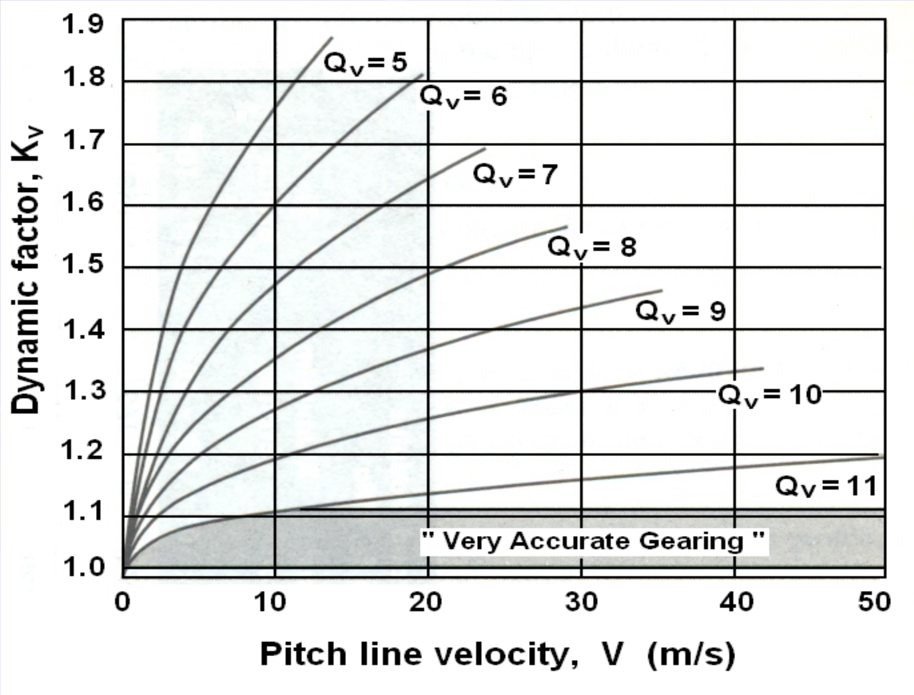

Fig.11.17 The dynamic factors for the helical gear tooth

K_o = Overload factor which reflects the degree of non-uniformity of driving and load torques. It is given in Table 11.1

K_m = Load distribution factor which accounts for non uniform spread of the load across the face width. It depends on the accuracy of mounting, bearings, shaft deflection and accuracy of gears. Taken from Table 11.2.

Table 11.1 -Overload factor K_o

	Driven Machinery		
Source of power	Uniform	Moderate Shock	Heavy Shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Table 11.2 Load distribution factor K_m

	Face width (mm)			
Characteristics of Support	0 - 50	150	225	400 up
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.2	1.3	1.4	1.7
Less rigid mountings, less accurate gears, contact across the full face	1.5	1.6	1.7	2.0
Accuracy and mounting such that less than full-face contact exists	Over 2.0	Over 2.0	Over 2.0	Over 2.0

HELICAL GEAR – PERMISSIBLE TOOTH BENDING STRESS (AGMA)

Fatigue strength of the material is given by:

$$\sigma_e = \sigma_e' \, k_L \, k_v \, k_s \, k_r \, k_T \, k_f \, k_m \qquad (11.23)$$

Where, σ_e' endurance limit of rotating-beam specimen

k_L = load factor, = 1.0 for bending loads

k_v = size factor, = 1.0 for m < 5 mm and

= 0.85 for m > 5 mm

k_s = surface factor, is taken from Fig. 11.18 based on the ultimate strength of the material and for cut, shaved, and ground gears.

k_r = reliability factor, given in Table 11.3.

k_T = temperature factor, = 1 for T≤ 120°C

more than 120°C, k_T < 1 to be taken from AGMA standards

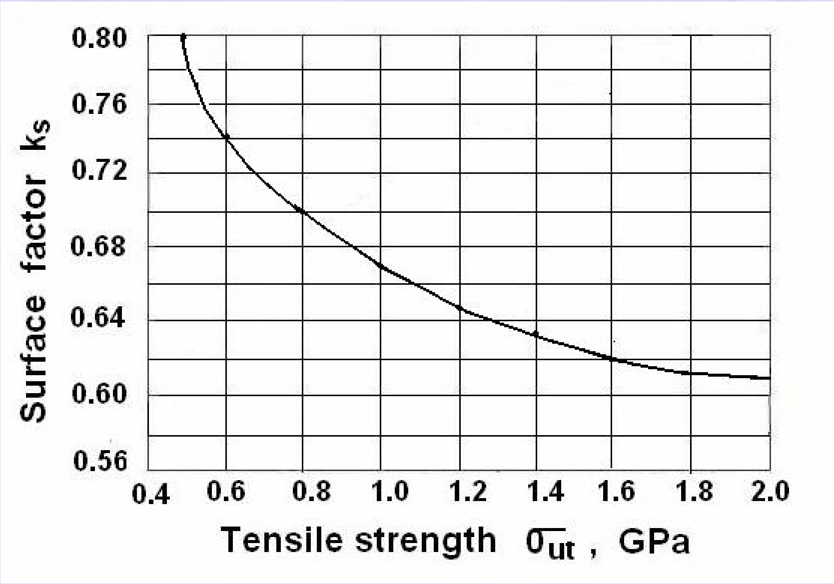

Fig.11.18 Surface factor k_s

Table 11.3 Reliability factor k_r

Reliability factor R	0.50	0.90	0.95	0.99	0.999	0.9999
Factor k_r	1.000	0.897	0.868	0.814	0.753	0.702

k_f = fatigue stress concentration factor. Since this factor is included in J factor, its value is taken as 1.

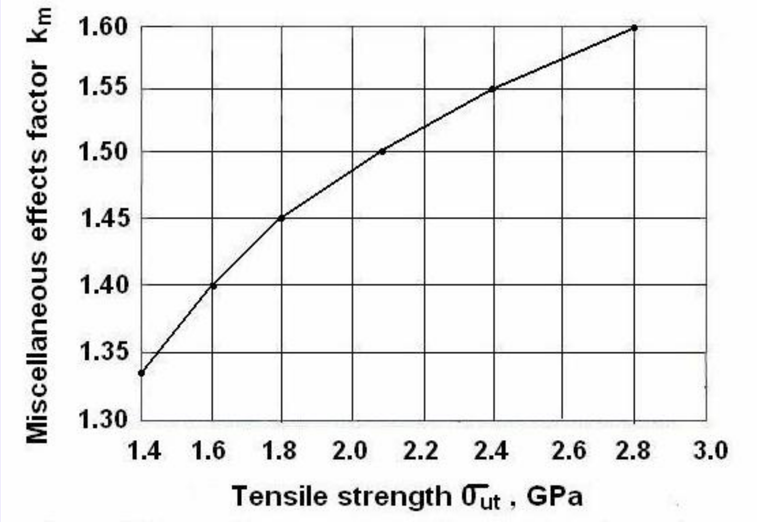

Fig. 11.19 Miscellaneous effects factor k_m

k_m = Factor for miscellaneous effects. For idler gears subjected to two way bending, = 1. For other gears subjected to one way bending, the value is taken from the Fig.5. Use k_m = 1.33 for σ_{ut} less than 1.4 GPa

Permissible bending stress

$$[\sigma] = \frac{\sigma_e}{s} \qquad (11.24)$$

Hence the design equation from bending consideration is: $\boxed{\sigma \le [\sigma] \qquad (11.25)}$

11.6 HELICAL GEAR - CONTACT STRESS

In the case of spur gears of contact ratio less than 2, the theoretical length of tooth contact is 1.0b.

With helical gears, the length of contact per tooth is b/cosψ and the helical action causes the total length of tooth contact to be approximately b/cosψ times the contact ratio (CR) at all times.

The AGMA recommends that 95% of this value be taken as the length of contact when computing contact stress.

The contact stress equation is given as

$$\sigma_H = C_p \sqrt{\frac{F_t}{bdI}\left(\frac{\cos\psi}{0.95CR}\right) K_v K_o (0.93K_m)} \qquad (11.26)$$

Elastic coefficient factor Cp is given by

$$C_p = 0.564 \sqrt{\frac{1}{\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}}} \qquad (11.27)$$

Where E and μ are the young's modulus and Poisson's ratio. Suffix 1 is for pinion and 2 is for gear. The values are given in Table 11.4

Table 11.4 Elastic coefficient C_p for spur gears and helical gears, in $\sqrt{MPa}$

Pinion Material (μ = 0.3 in all cases)	Gear Material			
	Steel	Cast Iron	Al Bronze	Tin Bronze
Steel, E = 207 GPa	191	166	162	158
Cast Iron, E = 131 GPa	166	149	149	145
Al Bronze, E=121 GPa	162	149	145	141
Tin Bronze, E=110 GPa	158	145	141	137

The geometry factor I given by:

$$I = \frac{\sin\varphi\cos\varphi}{2}\,\frac{i}{i+1} \qquad (11.28)$$

Where the speed ratio i $=n_1/n_2 = d_2/d_1$ and Ø is the transverse contact angle.

K_v , K_o and K_m as taken for bending stress calculation.

The contact ratio is given by:

$$CR = \left(\frac{\sqrt{(r_1 + a)^2 - r_{b1}{}^2} + \sqrt{(r_2 + a)^2 - r_{b2}{}^2} - (r_1 + r_2) \sin\varphi}{\pi m \cos\varphi} \right) \quad (11.29)$$

Where r is the pitch circle radius, r_b is the base circle radius, suffix 1 for pinion and 2 for gear. a is the addendum, Ø is the transverse pressure angle.

HELICAL GEAR – SURFACE FATIGUE STRENGTH

Surface fatigue strength of the material is given by:

$\sigma_{sf} = \sigma_{sf}' K_L K_H K_R K_T$ (11.30)

Where

σ_{sf}' = surface fatigue strength of the material given in Table 11.5

K_L = Life factor given in Fig.11.20

Table 11.5 Surface fatigue strength σ_{sf}' (MPa) for metallic spur gears (10^7 cycles life with 99% reliability and temperature <120°C)

Material	σ_{sf}' (MPa)
Steel	2·8 (Bhn) – 69 MPa
Nodular Iron	0.95 [2·8 (Bhn) – 69 MPa]
Cast Iron , grade 20	379
Cast Iron , grade 30	482
Cast Iron , grade 40	551
Tin Bronze, AGMA 2C (11% Sn)	207
Aluminium Bronze (ASTM B 148 – 52) (Alloy 9C – H.T.)	448

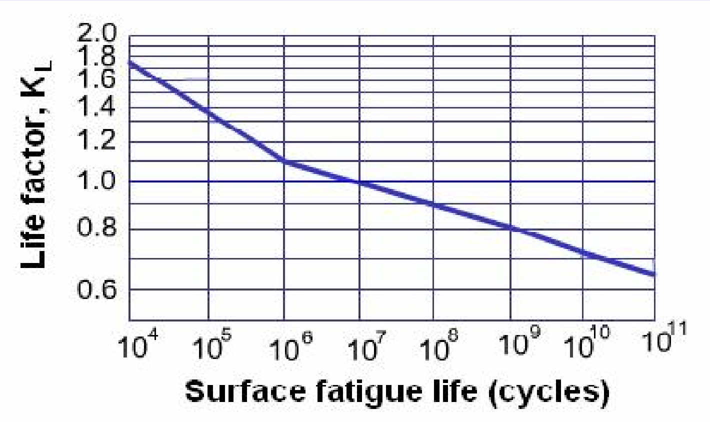

Fig.11.20 Life Factor K_L

K_H = Hardness ratio factor, given in Fig.11.21.

K ratio of Brinell hardness of the pinion by Brinell hardness of the Gear. K_H = 1.0 for K < 1.2

K_R = Reliability factor, given in Table 11.6

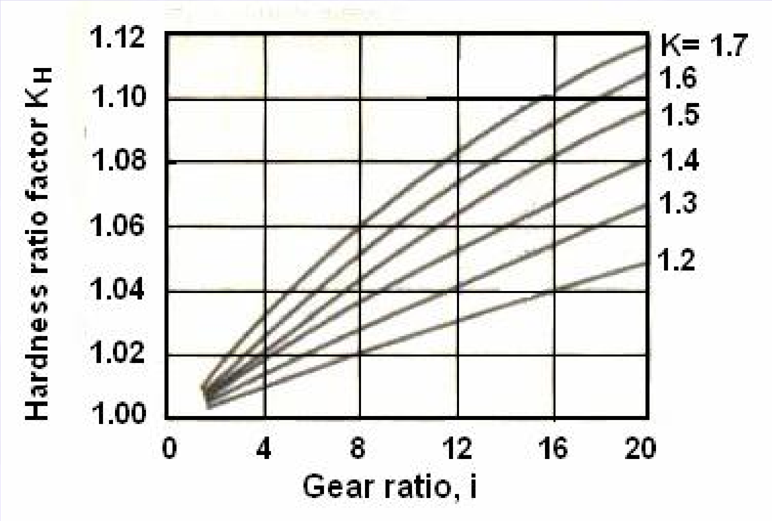

Fig.11.21 Hardness ratio factor, K_H
K= Brinell hardness ratio of the pinion and gear K_H = 1.0 for value of K below 1.2

Table 11.6 Reliability factor K_R

Reliability (%)	50	99	99.9
K_R	1.25	1.00	0.80

K_T = temperature factor,

= 1 for $T \leq 120^oC$, based on Lubricant temperature.

Above 120^oC, it is less than 1 to be taken from AGMA standards.

HELICAL GEAR – ALLOWABLE SURFACE FATIGUE STRESS (AGMA)

Allowable surface fatigue stress for design is given by

$$[\sigma_H] = \sigma_{Sf} / s \qquad (11.31)$$

Factor of safety s = 1.1 to 1.5

Hence Design equation is: $\boxed{\sigma_H \leq [\sigma_H] \qquad (11.32)}$

11.7 CROSSED HELICAL GEAR

a. Crossed helical gears are identical with other helical gears but are mounted on non-parallel shafts. They are basically non-enveloping worm gears since the gear blanks have a cylindrical form.

b. The relationship between the shaft angle and the helix angles of mating gears is

$\sigma = \psi_1 \pm \psi_2$ (11.33)

Where σ is the shaft angle. + sign is used when the gears have the same hand, and - sign when they are opposite hand.

c. Opposite hand crossed helical gears are used when the shaft angle is small.

d. The most common shaft angle is 90° that results in mating gears with complementary helix angles of the same hand.

e. The action of the crossed helical gears differs fundamentally from that of parallel helical gears in that the mating teeth slide across each other as they rotate.

f. The sliding velocity increases with increasing shaft angle.

g. For a given shaft angle, the sliding velocity is least when the two helix angles are the same.

h. Mating crossed helical gears must have the same p_n and $Ø_n$ but not necessarily the same transverse p and Ø.

i. The pitch diameter d is:

$$d = mZ = \frac{m_n Z}{\cos\psi} \qquad (11.34)$$

j. Furthermore, the velocity ratio is not necessarily the ratio of pitch diameters; it must be calculated as the ratio of the numbers of teeth.

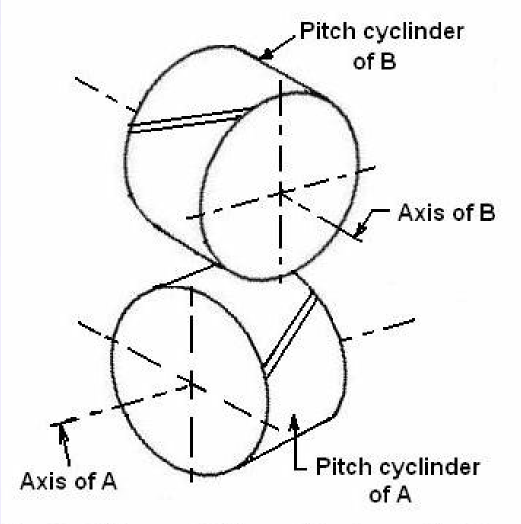

Fig. 11.22 View of the pitch cylinder of a pair of crossed –helical gear

11.7.1 CROSSED HELICAL GEAR - DESIGN TIPS

a. Crossed helical gears have very low load carrying capacities – usually less than a resultant tooth load of 400 N.

b. The limitation is one of surface deterioration, not bending strength.

c. Since they have point contact, to increase the load capacity contact ratios of 2 or more are usually used.

d. Low values of pressure angle and relatively large values of tooth depth are commonly specified to increase the contact ratio.

e. There are no standards for crossed helical gear tooth proportions. Many different proportions give good tooth action.
